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NONPERFECT CONTACT OF LAMINATED SHELLS WITH CONSIDERING DEBONDING BETWEEN LAMINAS IN TEMPERATURE FIELD

The problem of a heat conductivity of laminated shells through the heat-conducting layer in the case of debonding between laminas is considered here. The developed approach is based on our previous publications [1–4].

Let an elastic homogeneous anisotropic laminated shell of arbitrary geometry consists of Q layers with $2h^q$ thickness. There is a heat-conducting medium in the gap $h_0(\mathbf{x})$ between the laminas in the debonding area. The medium in the gap does not resist laminas deformation, and heat exchange between laminas is due to the thermal conductivity of the medium.

The thermodynamic state of the system, including the laminas and the heat-conducting medium is defined by the components of the stress and strain tensors and displacement vector, and the temperature and specific strength of the internal heat sources at the bodies and the medium, respectively.

The differential equations of thermoelasticity for the displacement vector components may be presented in the form

$$A_{ij}^{(q)} u_j^{(q)} + A_i^{(q)} \theta^{(q)} + b_i^{(q)} = 0\,, \quad A_{ij}^{(q)} = c_{ijkl}^{(q)} \partial_k \partial_l\,, \quad A_i^{(q)} = \beta_{ij}^{(q)} \partial_j \theta\,; \qquad (1)$$

where $\partial_i = \partial / \partial x_i$ are partial derivatives with respect to the space variables x_i; $c_{ijkl}^{(q)}$ are the elastic modulus; $\beta_{ij}^{(q)}$ are the linear thermal expansion coefficients.

The parts boundary conditions for displacements and traction have the form

$$p_i^{(q)} = \sigma_{ij}^{(q)} n_j = \psi_i^{(q)} \quad \left(\forall\, \mathbf{x} \in \partial V_p^{(q)}\right),$$
$$u_i^{(q)} = \varphi_i^{(q)} \quad \left(\forall\, \mathbf{x} \in \partial V_u^{(q)}, \forall\, \mathbf{x} \in V^{(q)}\right). \qquad (2)$$

In the area of debonding $\partial V_e^{(q)}$ mechanical boundary conditions has form of inequalities [5].

The equations of the shell heat conductivity can be written as

$$\lambda_{ij}^{(q)} \partial_i \partial_j \theta^{(q)} - \chi^{(q)} = 0 \quad \left(\forall\, \mathbf{x} \in V^{(q)}\right). \qquad (3)$$

Here $\lambda_{ij}^{(q)}$ are the coefficients of thermal conductivity.

The boundary conditions for the temperature and heat flux are

$$\theta^{(q)} = \theta_b^{(q)} \quad \left(\forall \mathbf{x} \in \partial V_\theta\right), \quad \mathbf{q}^{(q)} = \mathbf{q}_b^{(q)} \quad \left(\forall \mathbf{x} \in \partial V_q\right). \tag{4}$$

The temperature distribution in the heat-conducting medium is described by

$$\lambda_{ij}^* \partial_i \partial_j \theta_* - \chi_* = 0 \quad \left(\forall \mathbf{x} \in V^*\right). \tag{5}$$

Boundary conditions on the lateral sides of the heat-conducting medium are considered in the form

$$\lambda_{ij} \partial_n \theta^{(q)} + \beta_{ij} \left(\theta^{(q)} - \theta_b^{(q)}\right) = 0. \tag{6}$$

Conditions of heat conductivity in the contact surface between the heat-conducting medium and laminas have the form

$$\theta_* = \theta^{(q)}, \quad \lambda_{ij}^* \partial_n \theta_* = \lambda_{ij}^\alpha \partial_n \theta^{(q)} \quad \left(\forall \mathbf{x} \in \partial V_e^{(q)}\right). \tag{7}$$

In the area of debonding were close mechanical contact take place the thermal conditions can be rewritten as

$$q_\theta = \alpha_e \left(\theta^{(q)} - \theta_b^{(q)}\right) \quad \left(\forall \mathbf{x} \in \partial V_e^{(q)}\right), \tag{8}$$

where q_θ is the heat flux passing across the close mechanical contact area, α_e is the contacts thermal conductivity.

Analysis of the problem encounters mathematical difficulties caused by the dimension of the problem, as well as by its non-linearity. The problem can be partially simplified considering thin bodies, i.e. its dimension reducing obtained.

Let assume that the parameters, which describe the stress-strain state of each lamina as a three-dimensional body are sufficiently smooth functions of x_3 coordinate and may be expanded into Legandre's polynomial series. Then using the approach developed in [1–4, 6], they can be expressed as

$$u_i^{(q)}(\mathbf{x}) = \sum_{k=0}^{\infty} u_i^{(q)k}(\mathbf{x}_\alpha) P_k(\omega), \quad u_i^{(q)k}(\mathbf{x}_\alpha) = \frac{2k+1}{2h^{(q)}} \int_{-h^{(q)}}^{h^{(q)}} u_i^{(q)}(\mathbf{x}_\alpha, x_3) P_k(\omega) dx_3,$$

$$\theta^{(q)k}(\mathbf{x}_\alpha) = \frac{2k+1}{2h^{(q)}} \int_{-h^{(q)}}^{h^{(q)}} \theta^{(q)}(\mathbf{x}_\alpha, x_3) P_k(\omega) dx_3, \quad \theta^{(q)}(\mathbf{x}) = \sum_{k=0}^{\infty} \theta^{(q)k}(\mathbf{x}_\alpha) P_k(\omega),$$

$$\theta_*^{(q)}(\mathbf{x}) = \sum_{n=0}^{\infty} \theta_*^{(q)k}(\mathbf{x}_\alpha) P_k(\omega), \quad \theta_*^{(q)k}(\mathbf{x}_\alpha) = \frac{2k+1}{2h^{(q)}} \int_{-h^{(q)}}^{h^{(q)}} \theta_*^{(q)}(\mathbf{x}_\alpha, x_3) P_k(\omega) dx_3; \tag{9}$$

where $\omega = x_3 / h^{(q)}$ is a dimensionless coordinate.

The equations of thermoelasticity and heat conductivity and corresponding boundary conditions may be easily rewritten for coefficients in the Legandre's polynomial series. As result we obtain the 2-D equations and boundary conditions for k coefficient in the Legandre's polynomial.

In the first approximation, the shell theory considers only the first two terms of the Legandre polynomials series [6,8]. In this case the thermodynamic parameters, which describe the state of the laminated shell are introduced as

$$u_i^{(q)}(\mathbf{x})=u_i^{(q)0}(\mathbf{x}_\nu)+u_i^{(q)1}(\mathbf{x}_\nu)\,x_3/h\,,$$
$$\theta^{(q)}(\mathbf{x})=\theta^{(q)0}(\mathbf{x}_\nu)+\theta^{(q)1}(\mathbf{x}_\nu)\,x_3/h\,,$$
$$\theta_*^{(q)}(\mathbf{x})=\theta_*^{(q)0}(\mathbf{x}_\nu)+\theta_*^{(q)1}(\mathbf{x}_\nu)\,x_3/h\,. \tag{10}$$

Then the 2-D equations of thermo-elasticity (1) can be rewritten as

$$L_{ij}^{00}u_j^{(q)0}+L_{ij}^{01}u_j^{(q)1}+L_i^0\left(\theta^{(q)0}-\theta_0^{(q)0}\right)+b_i^{(q)0}=0\,,$$
$$L_{ij}^{10}u_j^{(q)0}+L_{ij}^{11}u_j^{(q)1}+L_i^1\left(\theta^{(q)1}-\theta_0^{(q)1}\right)+b_i^{(q)1}=0 \tag{11}$$

and the 2-D equations of heat-conductivity (5) become

$$\Delta_0\theta^{(q)0}+\frac{1}{2h}\left(Q_3^{(q)+}-Q_3^{(q)-}\right)+\left(k_1+k_2\right)Q_3^{(q)0}+\frac{\chi^{(q)0}}{\lambda_{(q)0}}=0\,,$$
$$\Delta_0\theta^{(q)1}+\frac{3}{2h}\left(Q_3^{(q)+}+Q_3^{(q)-}\right)+\left(k_1+k_2\right)Q_3^{(q)1}+\frac{\chi^{(q)1}}{\lambda_{(q)0}}=0\,. \tag{12}$$

Unknown parameters in equations (14) are

$$Q_3^{(q)+}-Q_3^{(q)-}=\frac{3}{4h}\left(\theta^{+}+T_k\right)+\frac{3\theta^{(q)0}}{2h}\,,\quad Q_3^{(q)0}=\frac{1}{2h}\left(\theta^{+}-T_k\right),$$
$$Q_3^{(q)+}+Q_3^{(q)-}=\frac{3}{2h}\left(\theta^{+}-T_k\right)-\frac{5\theta^{(q)1}}{2h}\,,\quad Q_3^{1}=\frac{3}{2h}\left(\theta^{+}+T_k\right)-\frac{3\theta^{1}}{2h}\,. \tag{13}$$

We will consider only one term in the Legendre polynomials series for θ_*. In this case equation of heat-conductivity in the gap is independent on the equations of thermo-elasticity (11) and heat-conductivity (12) of the laminate shell, see [1-3] for details.

To illustrate developed approach we consider axisymmetrical cylindrical shell which is in adhesive contact with cylindrical rigid body in the temperature field. There is a debonding area with gap $h_0(x)$ between shell and body. Let’s study the temperature field and stress-strain state using the approach presented above.

For simplicity we consider classic Kirchhoff-Love's theory of shells. In this case differential equations of thermo-elasticity and heat-conductivity for the axisymmetrical cylindrical shell have the form

$$\frac{d^4 w}{dx^4}+4\beta^4 w-\beta_0\theta^0-\beta_1\frac{d^2\theta^1}{dx^2}=\frac{1}{D}(p-q)\,,\quad \beta^4=\frac{3\left(1-\nu^2\right)}{4h^2r^2}\,; \tag{14}$$

$$\frac{d^2\theta^0}{dx^2}-\varepsilon_0^2\theta^0+F_0=0\,,\quad \frac{d^2\theta^1}{dx^2}-\varepsilon_1^2\theta^1+F_1=0\,; \tag{15}$$

where

$$F_0=0,5\varepsilon_0\left(\theta^-+T_k\right)+\frac{1}{2hr}\left(T_k-\theta^-\right)\,,\ \varepsilon_0=\frac{3}{h^2}\,,\quad \varepsilon_1=\frac{15}{h^2}\,,$$

$$F_1=0,5\varepsilon_1\left(T_k-\theta^-\right)+\frac{3}{2hr}\left(T_k+\theta^-\right)-\frac{3}{hr}\theta^1\,,\quad D=\frac{2Eh^3}{3\left(1-\nu^2\right)}\,,$$

$$T_k=\frac{\lambda_0\left(h_0-u_3\right)\left(3\theta^++6\theta^0-10\theta^1\right)+\lambda_* h\theta^-}{9\lambda_0\left(h_0-u_3\right)+\lambda_* h}\,.$$

The system of differential equations (14), (15) can be transformed into the integral equations of Hammerstein's type

$$\int_l G_\alpha(x,y)F_\alpha(y)dy=\theta^\alpha\,,$$

$$\int_l W(x,y)\left\{\frac{1}{D}[p(y)-q(y)]-\beta_0 F_3(y)\right\}dy=w\,, \tag{16}$$

where

$$F_3=\beta_1\left(F_1+\varepsilon_1^2\theta^1\right)-\beta_0\theta^0\,,\quad \beta_0=\frac{3(1-\nu)\alpha_\tau}{h^2 r}\,,\quad \beta_1=\frac{(1+\nu)\alpha_\tau}{h}\,.$$

The kernels in these integral equations are the Green's functions of the form

$$G_i(x,y)=\exp\left(-\varepsilon_i|x-y|\right)/2\varepsilon_i\quad (i=0,1)\,,$$

$$W(x,y)=\frac{1}{8\beta^3 D}\exp\left(-\beta|x-y|\right)\left[\cos\left(\beta|x-y|\right)+\sin\left(\beta|x-y|\right)\right]\,. \tag{17}$$

An algorithm for the problem solution has been elaborated in [1-3].

It is assumed that in homogeneous temperature field the shell is in unstressed state. Then stress σ_x and σ_θ in the axisymmetrical cylindrical shell are calculated by formulas

$$\sigma_x=\frac{E}{1-\nu^2}\left[\frac{d^2 w}{dx^2}z-(1+\nu)\alpha_t t_1\frac{z}{2h}\right], \tag{18}$$

$$\sigma_\theta = \frac{Ew}{r} - \alpha_t t_0 E + \frac{E}{1-\nu^2}\left[\nu \frac{d^2 w}{dx^2} z - (1+\nu)\alpha_t t_1 \frac{z}{2h}\right].$$

Using the Green's functions they can be presented in the integral form

$$\sigma_x(x) = b_0 \int_0^l T_0(\xi) \frac{d^2 G(\xi,x)}{dx^2} d\xi + b_1 \int_0^l G(\xi,x) T_1(\xi) d\xi - b_2 T_1(x) +$$

$$+ b_3 \int_0^l [p(\xi) - q(\xi)] \frac{d^2 G(\xi,x)}{dx^2} d\xi; \tag{19}$$

$$\sigma_\theta(x) = \nu \sigma_x(x) + \frac{Ew(x)}{r} - \alpha_t E\left[T_0(x) + T_1(x) z\right], \tag{20}$$

where

$$b_0 = \left[3E\alpha_t z r^3\right] / \left[(1+\nu) h^2 h_0\right],$$

$$b_1 = \left[3(1+\nu) E\alpha_\tau r^4 z\right] / \left[4h^3 h_0^2\right],$$

$$b_2 = \left[E\alpha_\tau r^2 z\right] / \left[(1-\nu) h h_0\right], \quad b_3 = \left[3r^4 z\right] / \left[2h^3 h_0\right]. \tag{21}$$

Calculation have been done for the data: temperature $h_0(x) = h_m \sin \pi x / l$, $\theta_0^+ = 750^\circ C$, $\theta^- = 0^\circ C$; geometrical parameters $r = 1{,}0\,m$, $h = 0{,}01\,m$, $h_0(x) = h_m \sin \pi x / l$, $h_m = 0{,}002\,m$, $l_d = 2r$; material properties: $\nu = 0{,}3$ $E = 2{,}5 \cdot 10^5\,MPa$, $\alpha_t = 2{,}5 \cdot 10^{-5}\ {}^\circ C^{-1}$, $\lambda_1 = 20 V / m^\circ C$, $\lambda_* = 10 V / m^\circ C$.

The temperature field and stresses σ_x and σ_θ distribution are presented in Fig. 1 and Fig. 2. The presented numerical results show that the debonding be-

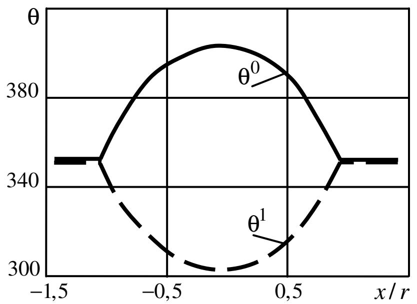

Fig. 1

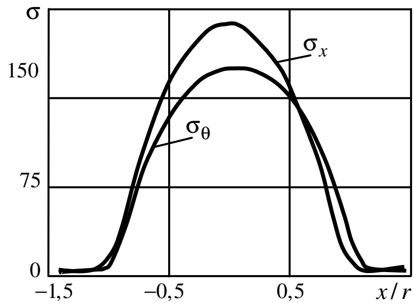

Fig. 2

tween laminas changes significantly the thermal conditions and effects the temperature field and stresses distribution.

Р Е З Ю М Е . Задача теплопровідності шаруватих оболонок через теплопровідний шар формулюється для випадку порушень суцільності між шарами. Підхід полягає у врахуванні зміни товщини шару за рахунок порушень суцільності та деформувань оболонок. Тривимірні рівняння термопружності та теплопровідності розкладаються у поліноміальні ряди Лежандра за товщиною. Рівняння першого наближення вивчено більш детально. Розглянуто числовий приклад теплопровідності шаруватої оболонки через теплопровідний шар.

S U M M A R Y . The problem of heat conductivity of laminated shells through the heat-conducting layer in the case of debonding between laminas is formulated. The approach consists in considering a change of layer thickness in the process of debonding and shells deformation. Three dimensional equations of thermoelasticity and heat conduction are expanded into a polynomial Legendre series in terms of the thickness. The first-approximations equations have been studied in more details. Numerical example of the heat conductivity of the laminated shells through the heat-conducting layer is considered.

LIST OF REFERENCES: 1. *Zozulya V. V.* The combined problem of thermoelastic contact between two plates though a heat conducting layer // J. Appl. Math. and Mech. – 1989. – Vol. 53, No 5. – P. 622–627. 2. *Zozulya V. V.* Contact cylindrical shell with a rigid body though the heat-conducting layer in transitional temperature field // *Mech. of Solids*. – 1991. – No 2. – P. 160–165. 3. *Zozulya V. V., Aguilar M.* Thermo-elastic contact and heat transfer between plates through the heat-conducting layer. - Heat transfer 2000, Eds. Sunden B., Brebbia C.A. Computational Mechanics Publications, Southampton, UK and Boston, USA, 2000. – P. 123–132. 4. *Zozulya V. V., Herrera-Franco P. J.* New model of laminated composites with considering unilateral contact and friction between laminas // Proc. of the 14th ASCE Eng. Mech. Division Conf.(CD-ROM), Austin, Texas, USA, May 21-24, 2000. – P. 343–348. 5. *Duvaut G., Lions J.-L.* Inequalities in mechanics and physics. – Berlin – New York: Springer–Verlag, 1976. – 403 p. 6. *Zozulya V. V.* The boundary integral equations for the shells of arbitrary geometry // Int. Appl. Mech. – 1998. – Vol. 34, No 5. – P. 79–83. 7. *Гуляев В. И., Баженов В. А., Лизунов П. П.* Неклассическая теория оболочек и её приложение к решению инженерных задач. – Львов: Вища школа, 1978. – 192 с.

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