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GENERAL SPATIAL DYNAMIC PROBLEM FOR AN ELLIPTIC CRACK UNDER THE ACTION OF A NORMAL SHEAR WAVE, WITH CONSIDERATION FOR THE CONTACT INTERACTION OF THE CRACK FACES

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Consideration is given to the contact interaction of the faces of a stationary plane elliptical crack under the action of a harmonic shear wave normally incident on the crack surface. The dependence of the mode II and III stress intensity factors on the wave number is studied for different values of the friction coefficient.

Keywords: spatial dynamic problem, plane elliptical crack, harmonic shear wave, crack faces, contact interaction, stress intensity factor, wave number, friction coefficient

Nowadays, the wide use of high-strength materials and the development of new and improvement of already available methods for analysis of local stresses make it possible to reduce considerably the safety factor, which, in turn, leads to a significant saving of weight of structures to be designed. However, if the stress–strain relationship was determined regardless of the possibility for cracks to occur and grow, then such an inaccurate stress–strain relationship in combination with a reduced safety factor might result in sudden collapse of structures under stresses much lower than the design maximum stress. Therefore, it is quite urgent to study the load-bearing capacity of structural materials with existing and incipient cracks under dynamic loading [2, 7–9, etc.]. During deformation, the opposite faces of cracks, which exist in any structural material, interact with each other, altering significantly the stress–strain distribution near the crack. That is why accounting for the contact interaction of crack faces should be an indispensable stage is solving problems in the dynamic fracture mechanics of cracked bodies. However, the overwhelming majority of studies conducted to date, except for those by the authors of the present paper [1, 3–6, 11–13, etc.], did not do that, though almost all authors pointed out the necessity of accounting for the contact of crack faces. The present paper sets out to solve a spatial dynamic problem for a stationary plane elliptic crack under the action of an arbitrarily polarized harmonic shear wave perpendicular to the crack surface and to account for the contact interaction of the crack faces.

Problem Formulation. Consider an elliptic crack with no initial opening under the action of a harmonic shear wave of frequency ω normally incident on the crack surface $\Omega = \{x_1^2 / a^2 + x_2^2 / b^2 \le 1, x_3 = 0\}$. The angle between the shear axis and the Ox_1-axis is γ (Fig. 1). The crack is located in a linearly elastic, homogeneous, isotropic material.

During a period of vibrations, the load caused by the incident wave is accompanied by contact forces for which the following unilateral constraints must be satisfied on the crack faces [1, 3–6, 12, 13]:

$$|\mathbf{q}_\tau(\mathbf{x},t)| \le k_\tau q_n(\mathbf{x},t) \Rightarrow \partial_t \Delta \mathbf{u}_\tau(\mathbf{x},t) = 0,$$

$$|\mathbf{q}_\tau(\mathbf{x},t)| > k_\tau q_n(\mathbf{x},t) \Rightarrow \partial_t \Delta \mathbf{u}_\tau(\mathbf{x},t) = -\mathbf{q}_\tau(\mathbf{x},t)\, |\partial_t \Delta \mathbf{u}_\tau(\mathbf{x},t) / \mathbf{q}_\tau|, \quad (1)$$

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where $q_n(\mathbf{x},t)$ and $\Delta q_\tau(\mathbf{x},t)$ are the normal and tangential components of the vector of contact forces; $\Delta \mathbf{u}_\tau(\mathbf{x},t)$ is the tangential component of the displacement discontinuity vector that characterizes the mutual displacements of the faces in the crack plane; and $k_\tau > 0$ is the friction coefficient. The distribution of the normal component of the contact force vector is assumed constant in time and over the crack surface.

Solution Method. During deformation, time-dependent adhesion (the opposite faces are retained by forces of friction) and sliding (the crack faces move with a velocity dependent on the vector of contact forces) regions, whose boundaries are not known, appear on the crack surface. Since the boundary conditions are nonlinear, which is an inherent feature of the class of problems being considered, the solution, though periodic, cannot be represented in terms of harmonic functions. Let us expand the stress–strain components into Fourier series [1, 3, 6]:

$$p_j(\mathbf{x},t)=\mathrm{Re}\left\{\sum_{k=-\infty}^{+\infty} p_j^k(\mathbf{x})e^{i\omega_k t}\right\}, \qquad \Delta u_j(\mathbf{x},t)=\mathrm{Re}\left\{\sum_{k=-\infty}^{+\infty} \Delta u_j^k(\mathbf{x})e^{i\omega_k t}\right\},$$

where $\omega_k = 2\pi k/T$, $j=1, 2$, and $p_j^k(\mathbf{x})=\dfrac{\omega}{2\pi}\displaystyle\int_0^T p_j(\mathbf{x},t)e^{-i\omega_k t}dt,\ \Delta u_j^k(\mathbf{x})=\dfrac{\omega}{2\pi}\displaystyle\int_0^T \Delta u_j(\mathbf{x},t)e^{-i\omega_k t}dt.$

For each $k=\overline{-\infty,+\infty}$, the Fourier coefficients are related by the system of boundary integral equations

$$\begin{cases} p_1^k(\mathbf{x})=-\int_\Omega F_{11}(\mathbf{x},\mathbf{y},\omega_k)\Delta u_1^k(\mathbf{y})d\Omega-\int_\Omega F_{12}(\mathbf{x},\mathbf{y},\omega_k)\Delta u_2^k(\mathbf{y})d\Omega, \\ p_2^k(\mathbf{x})=-\int_\Omega F_{21}(\mathbf{x},\mathbf{y},\omega_k)\Delta u_1^k(\mathbf{y})d\Omega-\int_\Omega F_{22}(\mathbf{x},\mathbf{y},\omega_k)\Delta u_2^k(\mathbf{y})d\Omega, \end{cases} \tag{2}$$

where the integral kernels $F_{ij}(\mathbf{x},\mathbf{y},\omega_k)$ are fundamental solutions from the dynamic theory of elasticity [5, 6, 12, 13].

To solve the problem numerically, we will use the boundary-element method with constant approximation of the stress–strain components on plane polygonal elements Ω_i, $i=\overline{1,N}$, into which the crack surface is partitioned. Then, according to (2), we obtain the following system of linear algebraic equations:

$$-\mathbf{F}^k\mathbf{U}^k=\mathbf{P}^k, \qquad k=\overline{-\infty,+\infty}, \tag{3}$$

where the square matrices $\mathbf{F}^k$ have the form

$$\mathbf{F}^k=\begin{bmatrix} \int_{\Omega_1} F_{11}(\mathbf{x}_1,\mathbf{y},\omega_k)d\Omega & \cdots & \int_{\Omega_N} F_{11}(\mathbf{x}_1,\mathbf{y},\omega_k)d\Omega & \int_{\Omega_1} F_{12}(\mathbf{x}_1,\mathbf{y},\omega_k)d\Omega & \cdots & \int_{\Omega_N} F_{12}(\mathbf{x}_1,\mathbf{y},\omega_k)d\Omega \\ \vdots & & \vdots & \vdots & & \vdots \\ \int_{\Omega_1} F_{11}(\mathbf{x}_N,\mathbf{y},\omega_k)d\Omega & \cdots & \int_{\Omega_N} F_{11}(\mathbf{x}_N,\mathbf{y},\omega_k)d\Omega & \int_{\Omega_1} F_{12}(\mathbf{x}_N,\mathbf{y},\omega_k)d\Omega & \cdots & \int_{\Omega_N} F_{12}(\mathbf{x}_N,\mathbf{y},\omega_k)d\Omega \\ \int_{\Omega_1} F_{21}(\mathbf{x}_1,\mathbf{y},\omega_k)d\Omega & \cdots & \int_{\Omega_N} F_{21}(\mathbf{x}_1,\mathbf{y},\omega_k)d\Omega & \int_{\Omega_1} F_{22}(\mathbf{x}_1,\mathbf{y},\omega_k)d\Omega & \cdots & \int_{\Omega_N} F_{22}(\mathbf{x}_1,\mathbf{y},\omega_k)d\Omega \\ \vdots & & \vdots & \vdots & & \vdots \\ \int_{\Omega_1} F_{21}(\mathbf{x}_N,\mathbf{y},\omega_k)d\Omega & \cdots & \int_{\Omega_N} F_{21}(\mathbf{x}_N,\mathbf{y},\omega_k)d\Omega & \int_{\Omega_1} F_{22}(\mathbf{x}_N,\mathbf{y},\omega_k)d\Omega & \cdots & \int_{\Omega_N} F_{22}(\mathbf{x}_N,\mathbf{y},\omega_k)d\Omega \end{bmatrix},$$

and the column vectors $\mathbf{U}^k$ and $\mathbf{P}^k$,

$$\mathbf{U}^k=\begin{Bmatrix} \Delta u_1^k(\mathbf{y}_1) \\ \cdots \\ \Delta u_1^k(\mathbf{y}_N) \\ \Delta u_2^k(\mathbf{y}_1) \\ \cdots \\ \Delta u_2^k(\mathbf{y}_N) \end{Bmatrix}, \qquad \mathbf{P}^k=\begin{Bmatrix} p_1^k(\mathbf{x}_1) \\ \cdots \\ p_1^k(\mathbf{x}_N) \\ p_2^k(\mathbf{x}_1) \\ \cdots \\ p_2^k(\mathbf{x}_N) \end{Bmatrix},$$

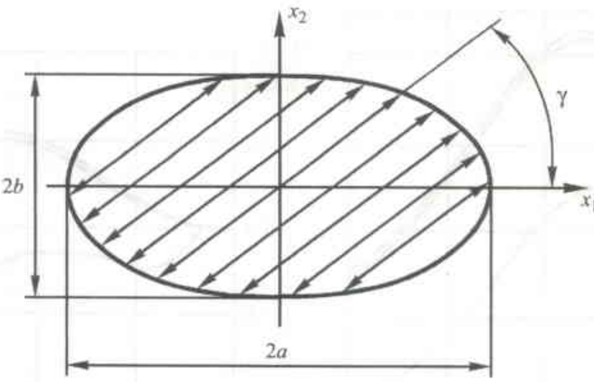

Fig. 1

the points $\mathbf{x}_j$ and $\mathbf{y}_j$ being located at the center of the boundary element Ω_j.

Note that in computing elements of (3), the integrals of $F_{ij}(\mathbf{x},\mathbf{y},\omega_k)$ containing nonintegrable singularities whose order is greater than the dimension of the domain of integration should be treated in the sense of the Hadamard finite part [6, 10–13].

Analysis of the Numerical Solution. Let us discuss, as an example, the numerical solution for an elliptic crack ($a/b = 2$). The shear axis coincides with the Ox_2-axis ($\gamma = \pi/2$). The crack is located in a material with the following properties: elastic modulus $E = 200$ GPa, Poisson's ratio $\nu = 0.25$, and density $\rho = 7800$ kg/m^3.

A major stage in solving problems in the mechanics of cracked solids is an analysis of the distribution of stress intensity factors near the crack tips.

The static mode II and III stress intensity factors for a plane elliptic cut can be calculated using the following expressions [7]:

$$K_{\mathrm{II}}^{\mathrm{stat}} = -\frac{\sqrt{\pi}}{(ab)^{3/2}\Pi^{1/4}}(aC\sin\beta + bB\cos\beta), \quad K_{\mathrm{III}}^{\mathrm{stat}} = \frac{\sqrt{\pi}(1-\nu)}{(ab)^{3/2}\Pi^{1/4}}(aB\sin\beta - bC\cos\beta),$$

$$\Pi = a^2\sin^2\beta + b^2\cos^2\beta, \quad B = -\frac{ab^2k^2\tau\cos\gamma}{(k^2-\nu)E(k) + \nu k'K(k)},$$

$$C = -\frac{ab^2k^2\tau\sin\gamma}{(k^2+\nu b^2a^{-2})E(k) - \nu b^2a^{-2}K(k)}, \quad k^2 = 1 - b^2/a^2, \quad k' = b/a, \quad a > b, \tag{4}$$

where $x_1 = a\cos\beta$ and $x_2 = b\sin\beta$ are the ellipse equations and $K(k)$ and $E(k)$ are the complete normal Legendre elliptic integrals of the first and second kinds, respectively,

$$K(k) = \int_0^{\pi/2}\frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}, \quad E(k) = \int_0^{\pi/2}\sqrt{1-k^2\sin^2\theta}\,d\theta.$$

The boundary-element method allows us to determine the values of stresses and displacement discontinuity on boundary elements approximating the crack surface. The values obtained on boundary elements located near the crack tips can be used to compute the stress intensity factors [1, 5, 6, 11–13].

Figures 2 and 3 show the maximum (in time) dynamic mode II and III stress intensity factors near the minor ($\beta = \pi/2$) and major ($\beta = 0$) vertices of the ellipse, respectively. The results obtained for different values of the reduced wave number k_2a are normalized to the corresponding static values (4). Curve *1* neglects the frictions between the crack faces, and curves *2* and *3* have been drawn for the following friction coefficients, respectively: $k_\tau = 0.02$ and $k_\tau = 0.3$.

The results obtained for a comparatively small friction coefficient (curves *2*) differ insignificantly from the results obtained regardless of the contact interaction of crack faces (curves *1*). Note that the shear stress intensity factors in the problem

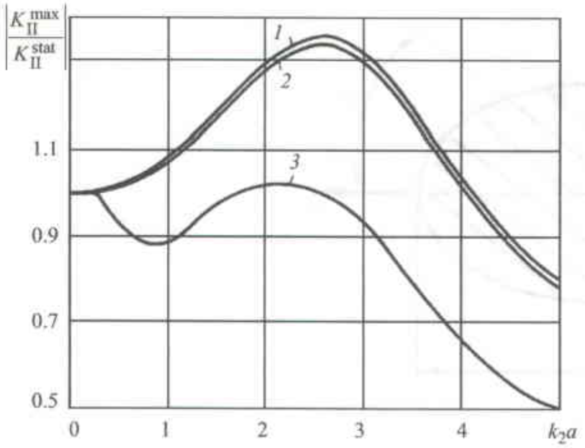

Fig. 2

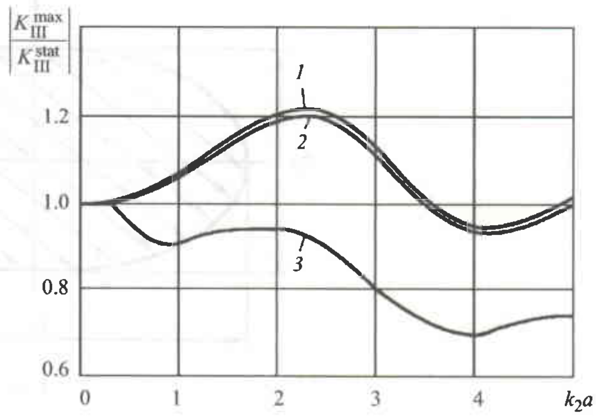

Fig. 3

for a circular crack reach their maxima for much smaller wave numbers ($k_2 a \approx 1.5$) [6, 12, 13]. A further increase in the friction coefficient leads to a gradual decrease in the mode II and III stress intensity factors; and beginning with some value of the friction coefficient, the static stress intensity factors exceed the dynamic stress intensity factors over the entire range of wave numbers (curves *3*).

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